

1.2.20. A rectangle with altitude x is inscribed in a triangle ABC with the base b and altitude h . Express the perimeter P and area S of the rectangle as functions of x .

Solution:

Let $PQ = RS = l$

Area of $\Delta ABC = \frac{bh}{2}$

Area of trapezium $PBCQ$

$$= \frac{1}{2} (\text{sum of parallel sides}) \times \text{height}$$

$$= \frac{1}{2} (l + b) x$$

Area of $\Delta APQ = \frac{1}{2} l(h-x)$

\therefore Area $\Delta ABC =$ Area $\Delta PBCQ +$ Area ΔAPQ

$\therefore \frac{bh}{2} = \frac{1}{2} (l+b)x + \frac{1}{2} l(h-x)$

$\Rightarrow l = b \left(1 - \frac{x}{h}\right)$

\therefore (S) Area of rectangle $PQRS = lx$

$\Rightarrow S = b \left(1 - \frac{x}{h}\right) x$ *Answer*

Perimeter $P = 2(l+x) = 2 \left(b - \frac{bx}{h} + x \right)$

$P = 2 \left(b + x \left(1 - \frac{b}{h}\right) \right)$ *Answer*

